Robust Sampled-Data Consensus-Based Cooperative Control of Multi UAVs

M. Di Ferdinando, P. Pepe, S. Di Gennaro

Abstract— **In this paper, the sampled–data formation control problem of multi unmanned aerial vehicles (UAVs), affected by actuation disturbances and observation errors, over strongly directed networks is studied. In particular, a robust consensus based sampled–data protocol enabling a connected and leaderless swarm of UAVs to reach a desired formation in altitude and position is proposed. The provided sampled–data control algorithm relies on the architecture of leaderless consensus, in which each UAV communicates with the adjacent UAVs only and, furthermore, on the input-to-state redesign methodology which is used in order to attenuate the effects of any bounded actuation disturbance and any bounded observation error. A sampled–data control strategy for the collision avoidance of adjacent UAVs is also included. The theory of the stabilization in sample–and–hold sense is used as a tool in order to prove that the sampled–data leaderless consensus of the multi UAVs swarm is ensured, regardless of the above disturbances and errors provided that the observation errors do not affect or affect marginally the new added control term. Actuator disturbances and observation errors are assumed to be bounded with arbitrarily large *a-priori* known bounds. Simulations show the good performances of the proposed robust sampled–data control algorithm.**

I. INTRODUCTION

In the last years, the cooperative control problem of autonomous robot has received a great attention by the researchers. In this context, one of the main field of interest concerns the development of control strategies for the cooperative coordination of multi unmanned aerial vehicles (UAVs). The coordinated use of multi UAVs has proved to be very helpful in many applications leading to an increasing demand by several areas such as military, logistic and farming just to mention a few. The growing interest on this topic is mainly due to the fact that a swarm of UAVs is able to accomplish complex tasks which cannot be feasible by a single UAV (see, for instance, [31], [30], [26]).

One of the most investigated coordination problem of multi UAVs concerns the formation control problem which can be formulated as follows: design a control strategy so that a set of UAVs is forced to fly in a desired formation while the team motion proceeds. The formation control problem of multi UAVs has been studied in the literature by the use of many approaches such as the consensus approach (see, for instance, [10], [11], [13], [14], [15], [19], [20], [29], [32], [33], [34], [37], [36]), the leader–follower approach ([16], [35], [1], [27]), the virtual structure–based approach ([28], [23], [39], [40]) and the behavior–based approach ([18]). In practical applications with swarms of UAVs, it is well known that the measurements required by the control strategy are sampled and often affected by errors. Moreover, disturbances affecting the proposed formation controller are often unavoidable in real practice. In the context of sampled-data control, results concerning the formation control problem of multi UAVs are very few. For instance in [3], a consensus–based formation controller for the deployment of multi-UAV systems in a distributed time-varying set-up is designed by modeling the dynamics of each UAV as a discrete-time integrator. The efficacy of the proposed control algorithm is verified with simulations and no stability proof is provided. In [22], an event-triggered controller for the time-varying formation problem of multi UAVs is proposed. Theoretical results are provided without take into account measurement errors and actuation disturbances. To our best knowledge, results concerning the robust sampled–data formation control of multi UAVs have never been provided in the literature. In particular, in the context of the sampled–data control, the problem of arbitrarily reducing the effect of an arbitrarily large actuator disturbance, as well as of an arbitrarily large observation error, has never been addressed in the literature concerning the sampled–data formation control of multi UAVs.

In this paper, a robust sampled–data controller for the formation control problem of multi UAVs is provided by a consensus–based approach and the theory of the stabilization in the sample–and–hold sense (see [5], [6]). In particular, a sampled–data controller for multi UAVs is provided in order to maintain a desired formation geometry while the swarm motion proceeds in rectilinear paths. The input–to–state (ISS) redesign methodology is used in order to attenuate the effects of any bounded actuation disturbance and any bounded observation error, as long as this observation error does not affect, or affects marginally, the new added state feedback. The results in [5], [6], concerning the theory of the robust stabilization in sample–and–hold sense applied in the consensus context, are used in order to prove that: there exists a suitably small sampling period such that the formation agreement is ensured in a semi–global practical sense, with

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arbitrarily small final formation tracking error, for the related sampled–data closed–loop multi UAVs system affected by the above observation errors and actuation disturbances. Moreover, the proposed control strategy allows also the collision avoidance of adjacent UAVs. The proposed robust sampled–data algorithm rely on the leaderless architectures in which each UAV interacts with the adjacent agents only and on the well–known emulation approach very used in the literature of sampled–data control (see, for instance, [4], [5], [6], [8], [9], [21], [24], [25]). The repellent potential approach (see [2], [12], [36]) is used for the design of the controller term related to the collision avoidance. To our best knowledge, it is the first time in the literature that theoretical results concerning a robust sampled–data formation controller for multi UAVs are provided. In the provided results, time–varying sampling intervals are allowed and the stability of the intersampling system behavior is proved. Simulations are performed in order to validate the results.

Notations. $\mathcal{Z}$ is the set of nonnegative integer numbers, $\mathbb{N}$ is the set of natural numbers, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^\star$ denotes the extended real line $[-\infty, \infty]$, $\mathbb{R}^+$ denotes the set of nonnegative reals $[0, \infty)$. The symbol $\|\cdot\|$ stands for any $(1, 2, \cdots, \infty)$ norm of a real vector. For a given positive integer n and for a given vector $x \in \mathbb{R}^n$, the symbol ${}^i x$, $i = 1, \cdots, n$, denotes the i-element of the vector x. The symbol $\otimes$ denotes the Kronecker product. For a given positive integer n and a given positive real h, the symbol $\mathcal{B}_{n,h}$ denotes the subset $\{x \in \mathbb{R}^n \mid \|x\| \leq h\}$. For a given positive integer n, the symbol I_n denotes the identity matrix in $\mathbb{R}^{n\times n}$. For given positive integers n, m and for a given matrix $\mathcal{D} \in \mathbb{R}^{n\times m}$, the symbol $\mathcal{D}^+$ denotes the pseudoinverse matrix of $\mathcal{D}$.

II. Preliminaries and Problem Statement

Firstly, for the reader's convenience, we recall some notation and results concerning the graph theory which will be used for the analysis of the sampled–data consensus problem addressed in this paper (see [5], [17], [38]). Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a *digraph* of order N with finite nonempty set of nodes $\mathcal{V} = \{1, \cdots, N\}$, a set of direct edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ with cardinality M. A directed path in digraph $\mathcal{G}$ is a sequence of directed edges. A directed tree is a digraph in which, for the root i and any other node j, there exists exactly one directed path from i to j. A spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. The graph $\mathcal{G}$ is called *strongly connected* if any two distinct nodes can be connected via a directed path and *quasi-strongly connected* if it has a spanning tree. The incidence matrix $\mathcal{D} = \mathcal{D}(\mathcal{G})$ for a digraph is a $\{0, \pm1\}$-matrix with rows and columns indexed by the vertexes and edges of $\mathcal{G}$,

$$[\mathcal{D}]_{ik} = \begin{cases} 1 & \text{if } i \text{ is the initial node of edge } e_k, \\ -1 & \text{if } i \text{ is the final node of edge } e_k, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Notice that each column of $\mathcal{D}$ contains exactly two nonzero entries 1 and -1.

Lemma 1: (see [38]) Considering a strongly connected digraph $\mathcal{G}$, the pseudoinverse of the incidence matrix $\mathcal{D}^+$ exists.

In the proposed framework, the model of the swarm of UAVs, which will be exploited for the design of the robust sampled–data formation controller, is described by the following equations (see [10], [11], [36])

$$\begin{aligned} \dot{p}_{x,i}(t) &= u_{x,i}(t) + K_x, \\ \dot{p}_{y,i}(t) &= u_{y,i}(t) + K_y, \\ \dot{p}_{z,i}(t) &= u_{z,i}(t) + K_z, \quad i = 1, \cdots, N, \end{aligned} \tag{2}$$

where: $p_{x,i}$, $p_{y,i}$, $p_{z,i} \in \mathbb{R}$ are the positions of the i-UAV related to the x-axis, the y-axis and the z-axis, respectively; $u_{x,i}$, $u_{y,i}$, $u_{z,i} \in \mathbb{R}$ are the control inputs of the i-UAV; N is the number of UAVs involved in the swarm; K_x, K_y, $K_z \in \mathbb{R}$ are known constants related to the linear trajectory followed by the swarm. It is assumed that the incidence matrix related to the N–agents system is described by:

$$\mathcal{D} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & -1 \\ -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix} \in \mathbb{R}^{N\times N}. \tag{3}$$

Notice that, the digraph related to the incidence matrix (3) is strongly connected. Then, from Lemma 1, there exists the pseudoinverse matrix $\mathcal{D}^+$. In particular, there exists the pseudoinverse matrix of $(\mathcal{D}^T \otimes I_3) = \bar{\mathcal{D}}$. By letting

$$\begin{aligned} p(t) &= \begin{bmatrix} p_1(t) & p_2(t) & \cdots & p_N(t) \end{bmatrix}^T \in \mathbb{R}^{3N}, \\ p_i(t) &= \begin{bmatrix} p_{x,i}(t) & p_{y,i}(t) & p_{z,i}(t) \end{bmatrix}^T \in \mathbb{R}^3, \\ u(t) &= \begin{bmatrix} u_1(t) & u_2(t) & \cdots & u_N(t) \end{bmatrix}^T \in \mathbb{R}^{3N}, \\ u_i(t) &= \begin{bmatrix} u_{x,i}(t) & u_{y,i}(t) & u_{z,i}(t) \end{bmatrix}^T \in \mathbb{R}^3, \quad i = 1, \cdots, N, \end{aligned} \tag{4}$$

the system (2) can be rewritten in compact form, as follows:

$$\dot{p}(t) = K + u(t), \tag{5}$$

where $K = \begin{bmatrix} K_x \\ K_y \\ K_z \end{bmatrix} \otimes b$ with $b = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^N$. Taking into account (7), in the proposed framework, the formation control problem of multi UAVs is studied from the point of view of desired inter-UAV distances. In particular, chosen a desired inter-UAV distance $z_{\text{ref}} = \begin{bmatrix} z_{\text{ref},1} & \cdots & z_{\text{ref},N} \end{bmatrix}^T \in \mathbb{R}^{3N}$, $z_{\text{ref},i} \in \mathbb{R}^3$, $i = 1, \cdots, N$, the formation control problem addressed in this paper can be summarized as follows:

$$\begin{aligned} &\lim_{t\to+\infty} \|p_i(t) - p_{i+1}(t) - z_{\text{ref},i}\| = 0, \quad i = 1, \cdots, N-1, \\ &\lim_{t\to+\infty} \|p_N(t) - p_1(t) - z_{\text{ref},N}\| = 0. \end{aligned} \tag{6}$$

Notice that, the condition (6) is equivalent to the reaching agreement.

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III. Main Results

In order to investigate the formation control problem 6, taking into account (3), (4), (5), let us now consider the new state variable

$$z(t) = \begin{bmatrix} z_1(t) \\ \vdots \\ z_N(t) \end{bmatrix} = \bar{\mathcal{D}}p(t) - z_{\text{ref}} = \\ = \begin{bmatrix} p_1(t) - p_2(t) \\ p_2(t) - p_3(t) \\ \vdots \\ p_{N-1}(t) - p_N(t) \\ p_N(t) - p_1(t) \end{bmatrix} - z_{\text{ref}}, \quad (7)$$

where: $z(t) \in \mathbb{R}^{3N}$; $z_i(t) \in \mathbb{R}^3$, $i = 1, 2, \cdots, N$. From (7), we obtain the following formation tracking error system

$$\dot{z}(t) = \bar{\mathcal{D}}\dot{p}(t) = \bar{\mathcal{D}}(K + u(t)) \\ = \bar{\mathcal{D}}u(t) = \begin{bmatrix} u_1(t) - u_2(t) \\ u_2(t) - u_3(t) \\ \vdots \\ u_{N-1}(t) - u_N(t) \\ u_N(t) - u_1(t) \end{bmatrix} = v(t), \quad (8)$$

with $v(t) = \bar{\mathcal{D}}u(t) \in \mathbb{R}^{3N}$ and $u_i \in \mathbb{R}^3$, $i = 1, 2, \cdots, N$. Notice that, from Lemma 1, taking into account (3), it is always possible to compute the protocol with respect to the original system (5) by the use of the relation

$$u(t) = \bar{\mathcal{D}}^{+}v(t). \quad (9)$$

The aims of this paper is to design a robust sampled–data controller for the system described by (8), which will be applied to the system (5) by the use of the relation (9) solving the reaching agreement (6). Firstly, in order to avoid possible collisions between swarm adjacent members, inspired by the repellent potential approach (see [2], [12], [36]), let us consider $\bar{k}_i : \mathbb{R}^{3N} \to \mathbb{R}^3$, $i = 1, \cdots, N$, be the functions defined, for $z \in \mathbb{R}^{3N}$, as follows

$$\bar{k}_i(z) = \\ = \begin{cases} \alpha\left(e^{-\beta(\|z_i + z_{\text{ref},i}\|)} - e^{-\beta r_s}\right)\bar{1}, & \|z_i + z_{\text{ref},i}\| \le r_s, \\ \bar{0}, & \text{otherwise}, \end{cases} \quad (10)$$

where: $\alpha, \beta > 0$ are control tuning parameters arbitrarily chosen; $r_s > 0$ is the security distance between the UAVs; $\bar{1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$; $\bar{0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$.

Let $\tilde{k}_i : \mathbb{R}^{3N} \to \mathbb{R}^3$, $i = 1, \cdots, N$, be the function defined, for $z \in \mathbb{R}^{3N}$, as follows

$$\tilde{k}_1(z) = \begin{cases} \begin{bmatrix} -2\alpha\,\text{sign}(^1z_1) \\ -2\alpha\,\text{sign}(^2z_1) \\ -2\alpha\,\text{sign}(^3z_1) \end{bmatrix}, & \text{if } \bar{k}_1(z) \neq \bar{0} \text{ or } \bar{k}_N(z) \neq \bar{0}, \\ \bar{0}, & \text{otherwise}, \end{cases}$$

$$\tilde{k}_i(z) = \begin{cases} \begin{bmatrix} -2\alpha\,\text{sign}(^1z_i) \\ -2\alpha\,\text{sign}(^2z_i) \\ -2\alpha\,\text{sign}(^3z_i) \end{bmatrix}, & \text{if } \bar{k}_i(z) \neq \bar{0} \text{ or } \bar{k}_{i-1}(z) \neq \bar{0}, \\ \bar{0}, & \text{otherwise}, \end{cases} \\ i = 2, \cdots, N. \quad (11)$$

Let $k_i : \mathbb{R}^{3N} \to \mathbb{R}^3$, $i = 1, \cdots, N$, be the functions defined, for $z \in \mathbb{R}^{3N}$, as follows

$$k_1(z) = -K_1 z_1 + \tilde{k}_1(z) + \bar{k}_1(z) + \bar{k}_N(z) \\ k_i(z) = -K_i z_i + \tilde{k}_i(z) + \bar{k}_i(z) + \bar{k}_{i-1}(z), \quad i = 2, \cdots, N, \quad (12)$$

where:

$$K_i = \begin{bmatrix} k_{i,x} & 0 & 0 \\ 0 & k_{i,y} & 0 \\ 0 & 0 & k_{i,z} \end{bmatrix}, \quad i = 1, \cdots, N, \quad (13)$$

with $k_{i,x}$, $k_{i,y}$, $k_{i,z} \in \mathbb{R}$, $i = 1, \cdots, N$, positive control tuning parameters arbitrarily chosen; $\bar{k}_i$, $\tilde{k}_i$, $i = 1, \cdots, N$, are the functions defined in (10), (11), respectively. The proposed static state feedback protocol $k : \mathbb{R}^{3N} \to \mathbb{R}^{3N}$ for the system (8) is described, for $z \in \mathbb{R}^{3N}$, as follows

$$k(z) = \begin{bmatrix} k_1(z) & k_2(z) & \cdots & k_N(z) \end{bmatrix}^T, \quad (14)$$

where k_i, $i = 1, \cdots, N$, are the functions defined in (12). Let P be the symmetric positive definite matrix described by

$$P = \epsilon I_{3N}, \quad (15)$$

where $\epsilon > 0$ is a parameter arbitrarily chosen. Let $S : \mathbb{R}^{3N} \to \mathbb{R}^{3N}$ be the function defined, for $z \in \mathbb{R}^{3N}$, as follows

$$S(z) = -2\rho(z^T P)^T, \quad (16)$$

where ρ is a further positive control tuning parameter. Notice that, the function $S(z)$ characterizes the new term to be added to the control law (14), in order to attenuate the effects of any bounded actuation disturbances and any bounded observation errors affecting the system at hand (see forthcoming Theorem 1). In particular, the function $S(z)$ is based on the ISS redesign methodology (see, for instance, [5] and references therein).

Before to state the main result of the paper, in the following the notion of partition of $[0, +\infty)$ is recalled [7], [24].

Definition 1: A partition $\pi = \{t_i,\ i = 0, 1, \cdots\}$ of $[0, +\infty)$ is a countable, strictly increasing sequence t_i, with $t_0 = 0$ and $t_i \to +\infty$ as $i \to +\infty$. The diameter of π, denoted $\text{diam}(\pi)$, is defined as $\sup_{i \ge 0} t_{i+1} - t_i$. The dwell time of π, denoted $\text{dwell}(\pi)$, is defined as $\inf_{i \ge 0} t_{i+1} - t_i$. For any positive real $\theta \in (0, 1]$, $\delta > 0$, $\pi_{\theta,\delta}$ is any partition π with $\theta\delta \le \text{dwell}(\pi) \le \text{diam}(\pi) \le \delta$.

The main result of the paper is given by the following statement and is based on the results provided in [5] applied to swarms of UAVs.

Theorem 1: Let $\theta \in (0, 1]$. Then, for any positive reals $\bar{d}_m$, $\bar{d}_a$, r, R, $0 < r < R$, there exists a positive real $\bar{\rho}$

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such that for any $\rho \geq \bar{\rho}$ there exist positive reals δ (upper bound of the sampling period), T (settling time) and C (overshoot), such that for any partition $\pi_{\theta,\delta}$, for any initial state $z^0 \in \mathcal{B}_{3N,R}$, for any sequence $d_m : \mathcal{Z} \to \mathbb{R}^{3N}$ (measurement error) and for any sequence $d_a : \mathcal{Z} \to \mathbb{R}^{3N}$ (actuation disturbance) satisfying for $j = 0, 1, \cdots,$

$$\begin{aligned} \|d_a(j)\| \leq \bar{d}_a, \quad \|d_m(j)\| \leq \bar{d}_m, \\ \sup_{z \in \mathcal{B}_{3N,C}} \|S(z + e(j)) - S(z)\| \leq \bar{d}_m, \end{aligned} \tag{17}$$

the solution of the sampled–data closed–loop system described by (8) with

$$\begin{aligned} &v(t) = k(z(t_j) + d_m(j)) + S(z(t_j) + d_m(j)) + d_a(j), \\ &\quad t_j \leq t < t_{j+1}, \quad j = 0, 1, \cdots, \end{aligned} \tag{18}$$

exists $\forall t \geq 0$ and, furthermore, satisfies:

$$\begin{aligned} \|z(t)\| \leq C, \qquad \forall t \geq 0, \\ \|z(t)\| \leq r, \qquad \forall t \geq T. \end{aligned} \tag{19}$$

Remark 1: In Theorem (1), it shows that for any bounded actuation disturbance and for any bounded observation error, there exist a suitable positive real $\bar{\rho}$ and a suitably small sampling period, such that, the trajectories of the related sampled–data closed–loop system (8)–(18), starting in any large ball of the origin of radius R, remain uniformly bounded and are driven in finite–time into any arbitrarily fixed small ball of the origin, and are kept in it thereafter. We highlight that, in Theorem 1, the actuation disturbances d_a and the observation errors d_m are unknown. The only required knowledges are the related (arbitrary as long as finite) upper bounds. Moreover, it is also required that the observation errors do not affect, or affect marginally the new added control term S (see (17)).

Remark 2: Notice that, from Theorem 1 and taking into account (9), we can conclude that the formation control problem (6) is solved in a semi–global practical sense with the robust sampled–data controller

$$\begin{aligned} &u(t) = \bar{\mathcal{D}}^{+}\Big(k(z(t_j) + d_m(j)) + S(z(t_j) + d_m(j)) + d_a(j)\Big), \\ &\quad t_j \leq t < t_{j+1}, \quad j = 0, 1, \cdots, \end{aligned} \tag{20}$$

regardless to any bounded actuation disturbance and to any bounded observation error.

Proof 1: In order to prove Theorem 1, thanks to the results proved in [5], we have to check that Assumption 2 in [5] holds for the system described by (8) in closed–loop with the protocol $v(t) = k(z(t))$ where k is the function defined in (14). In particular, we have to prove that there exists a control Lyapunov pair $V, W : \mathbb{R}^{3N} \to \mathbb{R}^{+}$ (see [7]), such that for any $z \in \mathbb{R}^{3N}$, the following holds:

$$\frac{\partial V}{\partial z}\dot{z}(t) \leq -W(z). \tag{21}$$

Taking into account the matrix P in (15), let $V, W : \mathbb{R}^{3N} \to \mathbb{R}^{+}$ be the functions defined, for $z \in \mathbb{R}^{3N}$, as

$$\begin{aligned} V(z) &= z^T P z, \\ W(z) &= 2\epsilon z^T(\lambda_{\min}(K) I_{3N})z, \end{aligned} \tag{22}$$

where: $K = \begin{bmatrix} K_1 & \bar{0} & \cdots & \bar{0} \\ \bar{0} & K_2 & \cdots & \bar{0} \\ \vdots & \vdots & \cdots & \vdots \\ \bar{0} & \bar{0} & \cdots & K_N \end{bmatrix} \mathbb{R}^{3N\times 3N}$, with K_i, $i = 1, \cdots, N$, the matrix defined in (13); $\bar{0} \in \mathbb{R}^{3\times 3}$ is the matrix with all zero elements. As far as inequality (21) is concerned, taking into account (8) with $v(t) = k(z(t))$ where k is the function defined in (14) and the function V, W defined in (22), for any $z \in \mathbb{R}^{3N}$, the following holds

$$\frac{\partial V}{\partial z}\dot{z}(t) = \frac{\partial V}{\partial z}k(z) \leq -2\epsilon\lambda_{\min}(K)|z|^2 = -W(z). \tag{23}$$

From (23), we can conclude that Assumption 2 in [5] here holds with the control Lyapunov pair V, W in (22). The proof of the theorem is complete.

IV. Simulations

In this section, an application of the provided results to a swarm of four UAVs is presented. In particular, let us consider the multi-agent system described by (2) with $N = 4$. In this case, the incidence matrix in is given by

$$\mathcal{D} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}. \tag{24}$$

In performed simulations, it has been chosen: $\alpha = 4$; $\beta = 5$; $r_s = 0.5[m]$; $k_{i,x} = k_{i,y} = k_{i,z} = 0.3$, $i = 1, \cdots, N$. The parameters of the robustification term S in (16) have been chosen as: $\epsilon = 0.2$; $\rho = 3.5$. The initial positions of the UAVs have been chosen equal to:

$$\begin{aligned} &p_1(0) = \begin{bmatrix} 0 & -3.2 & 1 \end{bmatrix}^T, \quad p_2(0) = \begin{bmatrix} -2 & -3 & 1.5 \end{bmatrix}^T, \\ &p_3(0) = \begin{bmatrix} -2 & -1 & 1.5 \end{bmatrix}^T, \quad p_4(0) = \begin{bmatrix} 1.2 & -2.2 & 1 \end{bmatrix}^T. \end{aligned} \tag{25}$$

The desired formation is chosen as:

$$\begin{aligned} &z_{\text{ref},1}(0) = \begin{bmatrix} -2 & 2 & 0 \end{bmatrix}^T, \quad z_{\text{ref},2}(0) = \begin{bmatrix} 2 & 2 & 0 \end{bmatrix}^T, \\ &z_{\text{ref},3}(0) = \begin{bmatrix} 2 & -2 & 0 \end{bmatrix}^T, \quad z_{\text{ref},4}(0) = \begin{bmatrix} -2 & -2 & 0 \end{bmatrix}^T. \end{aligned} \tag{26}$$

As far as the actuator disturbance is concerned, we consider

$$d_a(j) = 0.01\begin{bmatrix} \cos(0.5t_j + 1) + d_{a,1}(j) + 10.5 \\ \cos(0.5t_j + 1) + d_{a,2}(j) + 11 \\ \cos(0.5t_j + 1) + d_{a,3}(j) + 11.5 \\ \sin(0.5t_j + 1) + d_{a,4}(j) + 12 \\ \sin(0.5t_j + 1) + d_{a,5}(j) + 12.5 \\ \sin(0.5t_j + 1) + d_{a,6}(j) + 13 \\ 4\sin(0.5t_j + 1) + d_{a,7}(j) + 13.5 \\ 4\sin(0.5t_j + 1) + d_{a,8}(j) + 14 \\ 4\sin(0.5t_j + 1) + d_{a,9}(j) + 14.5 \\ 5\cos(0.5t_j + 1) + d_{a,10}(j) + 15 \\ 5\cos(0.5t_j + 1) + d_{a,11}(j) + 15.5 \\ 5\cos(0.5t_j + 1) + d_{a,12}(j) + 16 \end{bmatrix} \in \mathbb{R}^{12}, \tag{27}$$

with $d_{a,i}(j) \in \mathbb{R}$, $i = 1, \cdots, 12$, taken from the interval $[-2, 2]$, respectively, by emulation of the uniform probability density functions. Moreover, the observation error here considered is described by: $d_m(j) =$

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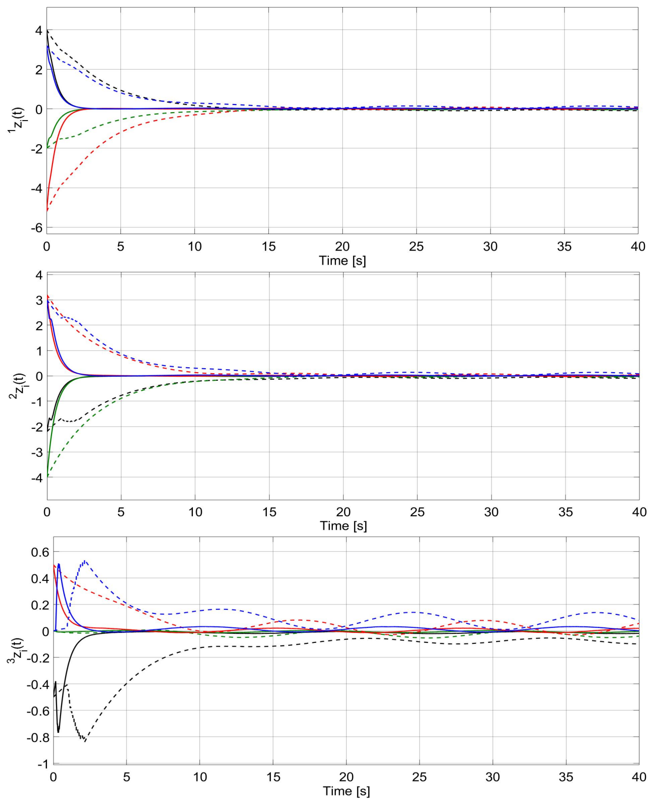

Fig. 1. Trajectories of the components ${}^1z_i$, ${}^2z_i$, ${}^3z_i$, $i = 1, \cdots, 4$, in the case of robustified (continuous–line) and non-robustified (dashed–line) controller.

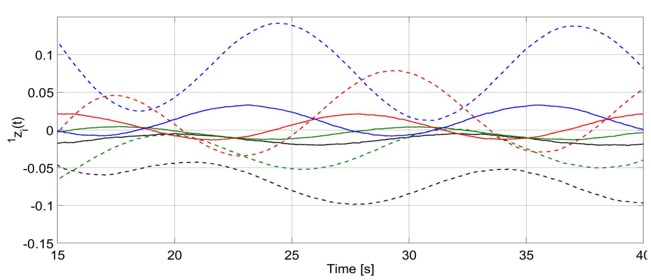

Fig. 2. Zoom of the component ${}^1z_i$, $i = 1, \cdots, 4$, in the case of robustified (continuous–line) and non-robustified (dashed–line) controller.

$\left[d_{m,1}(j) \quad \cdots \quad d_{m,12}(j)\right]^T$, where $d_{m,i}(j) \in \mathbb{R}$, $i = 1, \cdots, 12$, taken from the interval $[-0.02, 0.02]$, respectively, by emulation of the uniform probability density functions. The first addressed scenario concerns the case in which the swarm of UAVs has to reach a desired formation starting from a random initial condition (i.e. $K_x = K_y = K_z = 0$). In Figs. 1 and 2, the trajectory of $z(t)$, in both cases of robustified and non–robustified (i.e. $S(z) = 0$) controller with a sampling period equal to $\delta = 0.01$ is plotted. Figs. 1 and 2 clearly show the good performances of the robustified controller with respect to the non-robustified one. In particular, the robustified controller drastically attenuates the effects of the involved disturbances and forces the state variables to a neighbourhood of the origin which is much smaller than the one with the non-robustified controller. Fig. 3 shows the evolution of each UAV in the space. The second scenario addressed concerns the case in which the swarm of UAVs has to reach a desired formation starting from a random initial condition meanwhile a rectilinear reference path is followed. In particular, in this case $K_x = 1$, $K_y = K_z = 0$. In Fig. 3, the trajectory of the swarm of UAVs, in both cases of robustified and non–robustified (i.e. $S(z) = 0$) controller with a sampling period equal to $\delta = 0.01$ is plotted. Also in this case, the better perfomances of the robusified controller with respect to the non-robustified one are evident.

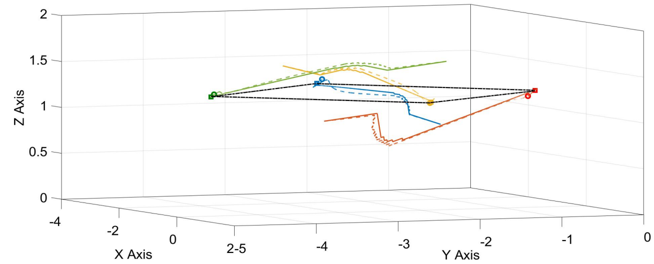

Fig. 3. Evolution of each UAV of the swarm in the case of robustified (continuous–line and square point) and non-robustified (dashed line and circle point) controller. In black dashed line the desired geometry of the formation is reported.

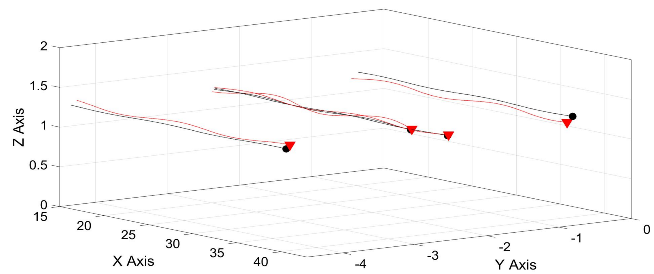

Fig. 4. Evolution of each UAV of the swarm in the case of robustified (black line) and non-robustified (red line) controller.

V. Conclusion

In this paper, a robust consensus based sampled–data protocol has been proposed for the formation control problem of multi unmanned aerial vehicles (UAVs), affected by actuation disturbances and observation errors, over strongly directed networks. In particular, a sampled–data controller for multi UAVs has been provided in order to achieve a desired formation geometry while the swarm motion proceeds in rectilinear paths. It has been proved that the proposed robust sampled-data protocol achieves the formation agreement in a semi–global practical sense, with arbitrarily small final geometry tracking error, of the related sampled–data closed–loop multi UAVs system, regardless of the above observation errors and actuation disturbances. In order to design the proposed sampled–data protocol, a leaderless consensus approach has been used. The proposed sampled–data protocol allows also the collision avoidance between adjacent UAVs

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of the swarm. Furthermore, the ISS redesign methodology has been used in order to attenuate the effects of any bounded actuation disturbance and any bounded observation error, as long as this observation error does not affect, or affects marginally, the new added state feedback. The stabilization in the sample–and–hold sense theory in the context of robust consensus control has been used as a tool in order to prove the results. Future investigations will concern the robust quantized sampled–data formation problem of multi UAVs with time–varying formation geometries.

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